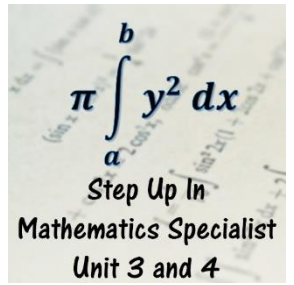


Polar Form

Problems Worksheet



1. Convert the following complex numbers. If stated in Cartesian form, convert to polar form $r \operatorname{cis} \theta$ where $r > 0$ and $-\pi < \theta \leq \pi$. If stated in polar form, convert to Cartesian form.

a. $z = -1 + i$

b. $z = 1 - \sqrt{3}i$

c. $z = 6 \cos \frac{\pi}{2} + 6i \sin \frac{\pi}{2}$

d. $z = 8 \operatorname{cis} \left(-\frac{\pi}{6} \right)$

e. $z = 2$

f. $z = -\pi i$

2. Calculate the exact distance between the points $z_1 = -2\sqrt{3} - 2i$ and $z_2 = \left[5, -\frac{\pi}{6} \right]$.

3. Let $z_1 = 2 \operatorname{cis} \frac{\pi}{3}$, $z_2 = 0.5 \operatorname{cis} \frac{5\pi}{6}$ and $z_3 = 2\sqrt{2} \operatorname{cis} \left(-\frac{2\pi}{3}\right)$. Complete the following multiplications and divisions of complex numbers using polar form. Give your answers in the form $r \operatorname{cis} \theta$ where $r > 0$ and $-\pi < \theta \leq \pi$.

a. $z_1 z_2$

b. $z_1 z_3$

c. $\frac{z_2 z_3}{i}$

d. $(iz_1)^2$

e. $\frac{iz_1}{z_3^2}$

4. Let $w_1 = 1 + \sqrt{3}i$ and $w_2 = -3 + 3i$.

a. Working in Cartesian form, determine $w_1 w_2$ and $\frac{w_1}{w_2}$.

b. Working in polar form, determine $w_1 w_2$ and $\frac{w_1}{w_2}$.

c. Determine $(w_1)^5$.

5.

a. Write $\frac{1+\sqrt{3}i}{1+i}$ in polar form.

b. Hence determine the exact value of $\cos \frac{\pi}{12}$.

6. Use de Moivre's theorem to determine an expression for $\sin 3\theta$ in terms of $\sin \theta$ only.

7. The general complex number $w = a + bi$ can be written in polar form $r \operatorname{cis} \theta$ where $r > 0$ and $0 < \theta < \frac{\pi}{2}$. Determine the magnitude and argument of each of the following in terms of r and θ .

a. $i^2 z$

b. $-a + bi$

c. $2a - 2bi$

8. Let $w_1 = r_1 \operatorname{cis} \alpha$, $w_2 = r_2 \operatorname{cis} \beta$ and $w_3 = r_3 \operatorname{cis} \theta$. Determine the following in terms of the modulus and/or the arguments of w_1 , w_2 and w_3 .

a. $|w_1 w_2 w_3|$

b. $\operatorname{arg} \left(\frac{w_1}{2w_2} \right)$

c. $\left| \frac{(w_2)^2}{3w_3} \right|$

d. $\operatorname{arg} \left(\frac{4w_3(w_2)^3}{(w_1)^2} \right)$