Polar Form

Problems Worksheet

$\pi \int_a^b y^2\, dx$

Step Up In Mathematics Specialist Unit 3 and 4

1. Convert the following complex numbers. If stated in Cartesian form, convert to polar form $r \operatorname{cis} \theta$ where $r > 0$ and $-\pi < \theta \leq \pi$. If stated in polar form, convert to Cartesian form.

   a. $z = -1 + i$

   b. $z = 1 - \sqrt{3}i$

   c. $z = 6\cos\frac{\pi}{2} + 6i\sin\frac{\pi}{2}$

   d. $z = 8\operatorname{cis}\left(-\frac{\pi}{6}\right)$

   e. $z = 2$

   f. $z = -\pi i$

2. Calculate the exact distance between the points $z_1 = -2\sqrt{3} - 2i$ and $z_2 = \left[5, -\frac{\pi}{6}\right]$.

3. Let $z_1 = 2\operatorname{cis}\frac{\pi}{3}$, $z_2 = 0.5\operatorname{cis}\frac{5\pi}{6}$ and $z_3 = 2\sqrt{2}\operatorname{cis}\left(-\frac{2\pi}{3}\right)$. Complete the following multiplications and divisions of complex numbers using polar form. Give your answers in the form $r\operatorname{cis}\theta$ where $r > 0$ and $-\pi < \theta \le \pi$.

   a. $z_1 z_2$

   b. $z_1 z_3$

   c. $\frac{z_2 z_3}{i}$

   d. $(iz_1)^2$

   e. $\frac{iz_1}{{z_3}^2}$

4. Let $w_1 = 1 + \sqrt{3}i$ and $w_2 = -3 + 3i$.

   a. Working in Cartesian form, determine $w_1 w_2$ and $\frac{w_1}{w_2}$.

   b. Working in polar form, determine $w_1 w_2$ and $\frac{w_1}{w_2}$.

   c. Determine $(w_1)^5$.

5.

a. Write $\frac{1+\sqrt{3}i}{1+i}$ in polar form.

b. Hence determine the exact value of $\cos\frac{\pi}{12}$.

6. Use de Moivre's theorem to determine an expression for $\sin 3\theta$ in terms of $\sin\theta$ only.

7. The general complex number $w = a + bi$ can be written in polar form $r \operatorname{cis} \theta$ where $r > 0$ and $0 < \theta < \frac{\pi}{2}$. Determine the magnitude and argument of each of the following in terms of r and θ.

   a. $i^2 z$

   b. $-a + bi$

   c. $2a - 2bi$

8. Let $w_1 = r_1 \operatorname{cis} \alpha$, $w_2 = r_2 \operatorname{cis} \beta$ and $w_3 = r_3 \operatorname{cis} \theta$. Determine the following in terms of the modulus and/or the arguments of w_1, w_2 and w_3.

   a. $|w_1 w_2 w_3|$

   b. $arg\left(\frac{w_1}{2w_2}\right)$

   c. $\left|\frac{(w_2)^2}{3w_3}\right|$

   d. $arg\left(\frac{4w_3(w_2)^3}{(w_1)^2}\right)$